Mining Data Streams (Part 1)

CS246: Mining Massive Datasets
Jure Leskovec, Stanford University
http://cs246.stanford.edu

Data Streams

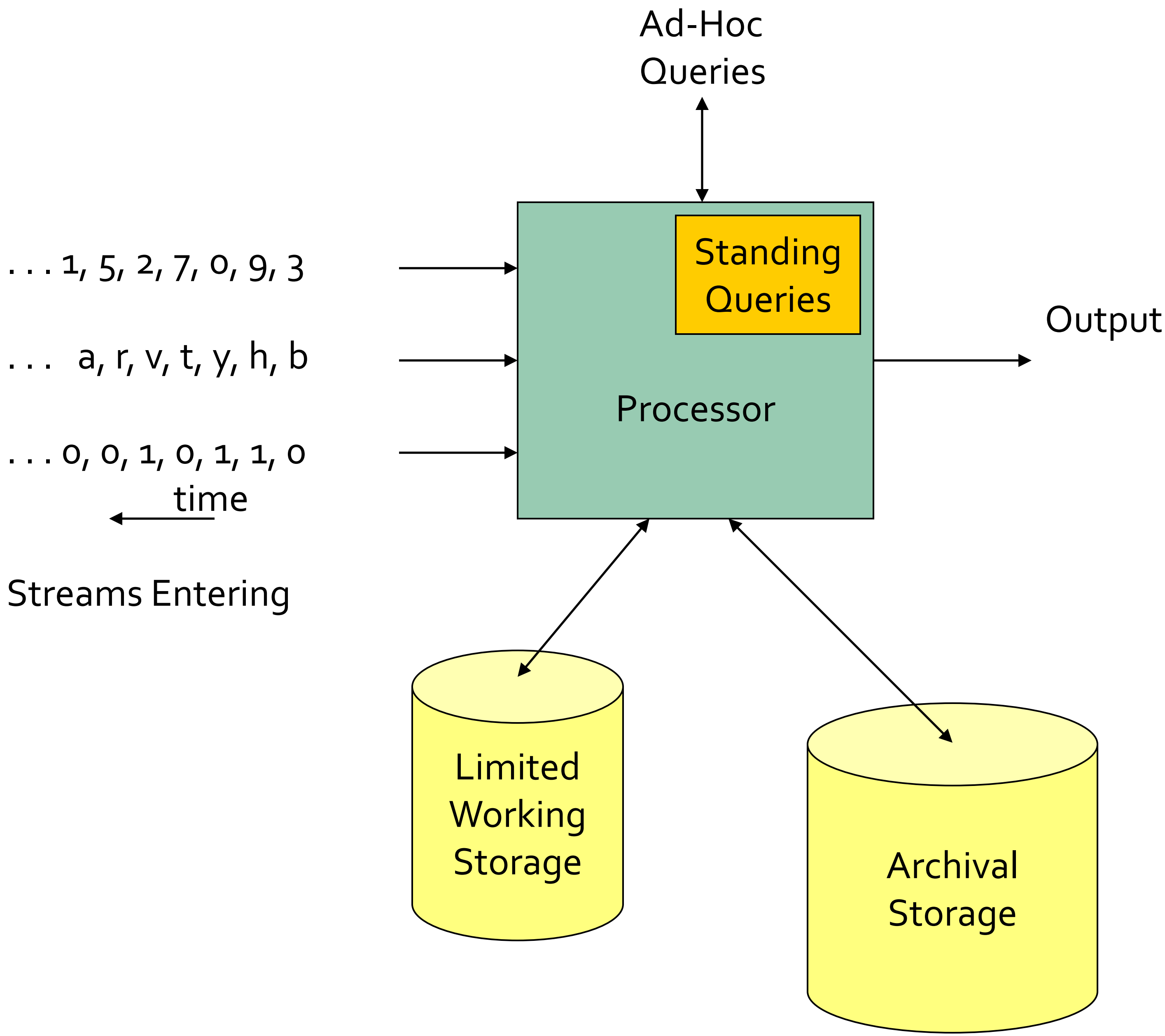
- In many data mining situations, we know the entire data set in advance

- Stream Management is important when the input rate is controlled externally:
  - Google queries
  - Twitter or Facebook status updates

The Stream Model

- Input tuples enter at a rapid rate, at one or more input ports
- The system cannot store the entire stream accessibly

- How do you make critical calculations about the stream using a limited amount of (secondary) memory?


Ad-Hoc
Queries
. . . 1, 5, 2, 7, 0, 9, 3
. . . a, r, v, t, y, h, b
. . . 0, 0, 1, 0, 1, 1, 0
time
Streams Entering
Standing
Queries
Processor
Output
Limited
Working
Storage
Archival
Storage

Applications – (1)

- Mining query streams
  - Google wants to know what queries are more frequent today than yesterday
- Mining click streams
  - Yahoo wants to know which of its pages are getting an unusual number of hits in the past hour
- Mining social network news feeds
  - E.g., Look for trending topics on Twitter, Facebook

Applications – (2)

- Sensor Networks
  - Many sensors feeding into a central controller
- Telephone call records
  - Data feeds into customer bills as well as settlements between telephone companies
- IP packets monitored at a switch
  - Gather information for optimal routing
  - Detect denial-of-service attacks

Problems on Data Streams

- Sampling data from a stream
- Filtering a data stream
- Queries over sliding windows
- Counting distinct elements
- Estimating moments (avg., std. dev., …)
- Finding frequent elements
- Frequent itemsets

Sampling from a Data Stream

- Since we can not store the entire stream, one obvious approach is to store a sample
- Two different problems:
  - Sample a fixed proportion of elements in the stream (say 1 in 10)
  - Maintain a random sample of fixed size over a potentially infinite stream

Sampling a Fixed Proportion

- Scenario: Search engine query stream
  - Stream of tuples: (user, query, time)
  - Answer questions such as: How often did a user run the same query on two different days?
  - Have space to store $1/10^{th}$ of query stream
- Naïve solution:
  - Generate a random integer in [0..9] for each query
  - Store query if the integer is 0, otherwise discard

Problem with Naïve Approach

- Consider the question: What fraction of queries by an average user are duplicates?
- Suppose each user issues s queries once and d queries twice (total of $s+2d$ queries)
  - Correct answer: $d/(s+2d)$
  - Sample will contain $s/10$ of the singleton queries and $2d/10$ of the duplicate queries at least once
  - But only $d/100$ pairs of duplicates
  - So the sample-based answer is: $d/(10s+20d)$

Solution: Sample Users

- Pick $1/10^{th}$ of users and take all their searches in the sample
- Use a hash function that hashes the user name or user id uniformly into 10 buckets

Generalized Solution

- Stream of tuples with keys:
  - Key is some subset of each tuple's components
    - e.g., tuple is (user, search, time); key is user
  - Choice of key depends on application
- To get a sample of size *a/b*:
  - Hash each tuple's key uniformly into *b* buckets
  - Pick the tuple if its hash value is at most *a*

Maintaining a fixed-size sample

- Suppose we need to maintain a sample of size exactly s
  - E.g., main memory size constraint
- Why? Don't know length of stream in advance
  - In fact, stream could be infinite
- Suppose at time t we have seen n items
  - Ensure each item is in the sample with equal probability s/n

Solution: Fixed Size Sample

- Store all the first *s* elements of the stream
- Suppose we have seen *n-1* elements, and now the n^{th} element arrives ($n > s$)
  - With probability s/n, pick the n^{th} element, else discard it
  - If we picked the n^{th} element, then it replaces one of the *s* elements in the sample, picked uniformly at random
- <u>Claim:</u> This algorithm maintains a sample with the desired property

Proof: By Induction

- Assume that after *n* elements, the sample contains each element seen so far with probability *s/n*
- When we see element *n+1*, it gets picked with probability *s/(n+1)*
- For elements already in the sample, probability of remaining in the sample is:

$$(1 - \frac{s}{n+1}) + (\frac{s}{n+1})(\frac{s-1}{s}) = \frac{n}{n+1}$$

Element n+1 discarded | Element n+1 not discarded | Element in the sample not picked

Sliding Windows

- A useful model of stream processing is that queries are about a *window* of length N – the N most recent elements received
- Interesting case: N is so large it cannot be stored in memory, or even on disk
  - Or, there are so many streams that windows for all cannot be stored

Sliding Window: 1 Stream

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

← Past   Future →

Counting Bits – (1)

- **Problem**:
  - Given a stream of 0's and 1's (WLOG)
  - Be prepared to answer queries of the form "how many 1's in the last k bits?" where $k \leq N$

- **Obvious solution**:
  Store the most recent N bits
  - When new bit comes in, discard the N +1st bit

0 1 0 0 1 1 0 1 1 1 0 1 0 1 0 $\boxed{1\ 1\ 0\ 1\ 1\ 0\ 1\ 1\ 0}$

← Past   Future →

Counting Bits – (2)

- You can not get an exact answer without storing the entire window
- Real Problem:
  What if we cannot afford to store N bits?
  - E.g., we're processing 1 billion streams and N = 1 billion
- But we are happy with an approximate answer

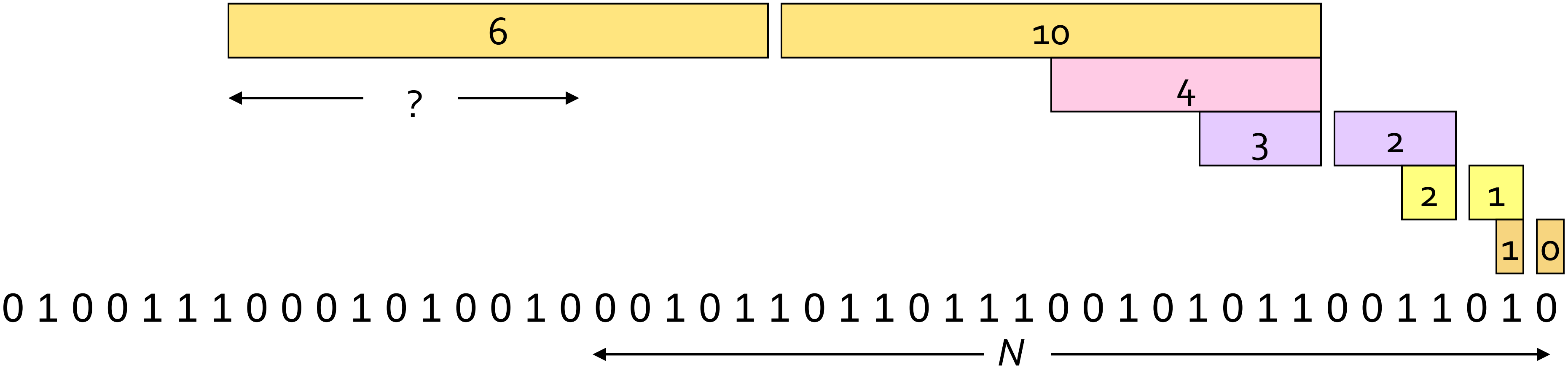
[Datar, Gionis, Indyk, Motwani]

DGIM Method

- Store O($\log^2 N$) bits per stream
- Gives approximate answer, never off by more than 50%
  - Error factor can be reduced to any fraction > 0, with more complicated algorithm and proportionally more stored bits

Idea: Exponential Windows

- Solution That Doesn't (Quite) Work:
  - Summarize exponentially increasing regions of the stream, looking backward
  - Drop small regions if they begin at the same point as a larger region



We can construct the count of the last N bits, except we are not sure how many of the last 6 are included.

What's Good?

- Stores only O($\log^2 N$) bits
  - O($\log N$) counts of $\log_2 N$ bits each
- Easy update as more bits enter
- Error in count no greater than the number of 1s in the "unknown" area

What's Not So Good?

- As long as the 1s are fairly evenly distributed, the error due to the unknown region is small – no more than 50%
- But it could be that all the 1s are in the unknown area at the end
- In that case, the error is unbounded

Fixup: DGIM method

- Instead of summarizing fixed-length blocks, summarize blocks with specific number of 1s.
  - Let the block "*sizes*" (number of 1s) increase exponentially

- When there are few 1s in the window, block sizes stay small, so errors are small

Timestamps

- Each bit in the stream has a *timestamp*, starting 1, 2, …
- Record timestamps modulo *N* (the window size), so we can represent any relevant timestamp in $O(\log_2 N)$ bits

Buckets

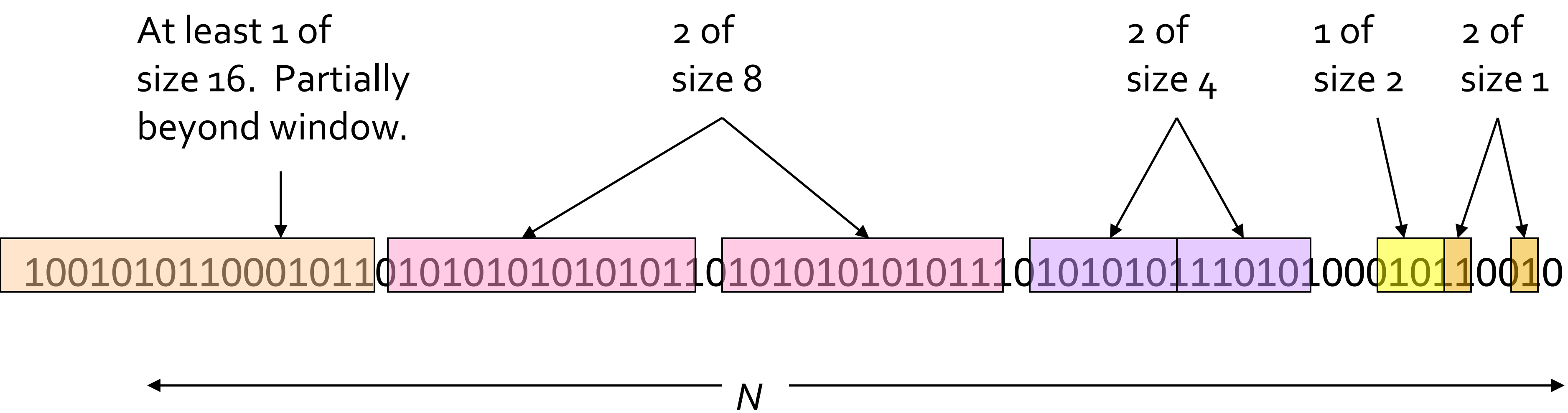
- A *bucket* in the DGIM method is a record consisting of:
  1. The timestamp of its end [O(log N) bits]
  2. The number of 1s between its beginning and end [O(log log N) bits]

- Constraint on buckets:
  Number of 1s must be a power of 2
  - That explains the O(log log N) in (2)26

Representing a Stream by Buckets

- Either one or two buckets with the same power-of-2 number of 1s
- Buckets do not overlap in timestamps
- Buckets are sorted by size
  - Earlier buckets are not smaller than later buckets.
- Buckets disappear when their end-time is > *N* time units in the past

Example: Bucketized Stream



Updating Buckets – (1)

- When a new bit comes in, drop the last (oldest) bucket if its end-time is prior to N time units before the current time

- If the current bit is 0, no other changes are needed

Updating Buckets – (2)

- If the current bit is 1:
  1. Create a new bucket of size 1, for just this bit
     - End timestamp = current time
  2. If there are now three buckets of size 1, combine the oldest two into a bucket of size 2.
  3. If there are now three buckets of size 2, combine the oldest two into a bucket of size 4.
  4. And so on …

Example: Updating Buckets

10010101100010110101010101010110101010101011101010101110101000101100010

00101011000101101010101010101101010101010111010101011101010001011001001

00101011000101101010101010101101010101010111010101011101010001011001001

0101100010110101010101010101101010101010111010101011101010001011001011001

0101100010110101010101010101101010101010111010101011101010001011001011001

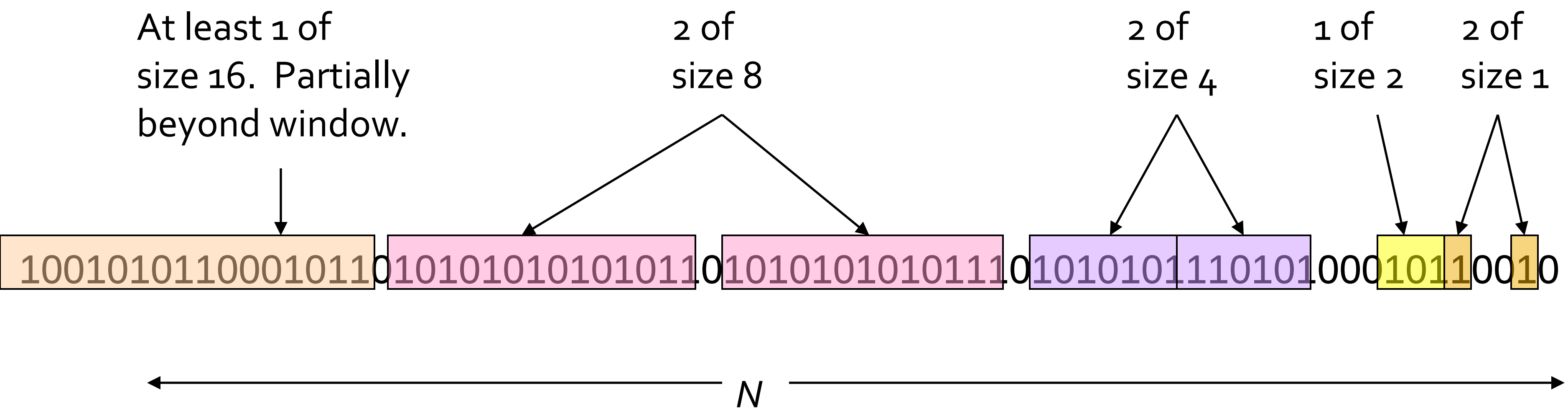
0101100010110101010101010101101010101010111010101011101010001011001011001

How to Query?

- To estimate the number of 1s in the most recent *N* bits:
  1. Sum the sizes of all buckets but the last
  2. Add half the size of the last bucket

- Remember: We do not know how many 1s of the last bucket are still within the window

Example: Bucketized Stream



Error Bound

- Suppose the last bucket has size 2^k.
- Then by assuming 2^{k-1} of its 1s are still within the window, we make an error of at most 2^{k-1}
- Since there is at least one bucket of each of the sizes less than 2^k, the true sum is at least $1 + 2 + 4 + .. + 2^{k-1} = 2^k - 1$.
- Thus, error at most 50%

Extensions

- Can we use the same trick to answer queries "How many 1's in the last k?" where $k < N$?
  - A: Find earliest bucket B that at overlaps with k. Number of 1s is the sum of sizes of more recent buckets + ½ size of B

- Can we handle the case where the stream is not bits, but integers, and we want the sum of the last k elements?

Reducing the Error

- Instead of maintaining 1 or 2 of each size bucket, we allow either r -1 or r for r > 2
  - Except for the largest size buckets; we can have any number between 1 and r of those
- Error is at most by 1/(r-1)
- By picking r appropriately, we can tradeoff between number of bits and error